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Super-allowed α decay above doubly-magic ^{100}Sn and properties of $^{104}\mathrm{Te} = {}^{100}\mathrm{Sn} \otimes \alpha$

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Abstract. α-decay half-lives for ^{104,105,106}Te and ^{108,109,110}Xe close above the doubly-magic ^{100}Sn are calculated from systematic double-folding potentials. The derived α preformation factors are compared to results for ^{212,213,214}Po and ^{216,217,218}Rn above the doubly-magic ^{208}Pb. α-decay energies of $Q_\alpha = 5.42 \pm 0.07$ MeV and 4.65 ± 0.15 MeV are predicted for ^{104}Te and ^{108}Xe; the corresponding half-lives are $T_{1/2} \approx 5$ ns for ^{104}Te and of the order of 60 μs for ^{108}Xe. Additionally, the properties of rotational bands in ^{104}Te are analyzed, and the first excited 2^+ state in ^{104}Te is predicted at $E_x = 650 \pm 40$ keV; it decays preferentially by γ emission with a reduced transition strength of 10 Weisskopf units to the ground state of ^{104}Te and with a minor branch by α emission to the ground state of ^{100}Sn.

PACS. 21.10.-k Properties of nuclei; nuclear energy levels – 21.10.Tg Lifetimes – 27.60.+j $90 \leq A \leq 149$ – 21.60.Gx Cluster models

1 Introduction

Studies of α-decay properties of nuclei with $Z \approx N$ in the mass region above $A \approx 100$ have been stimulated by recent experimental progress: Seweryniak *et al.* [1] have detected the α-decay of ^{105}Te at the Argonne fragment analyzer, and Liddick *et al.* [2] have analzed the α-decay chain $^{109}\mathrm{Xe}(\alpha)^{105}\mathrm{Te}(\alpha)^{101}\mathrm{Sn}$ at the recoil mass spectrometer of the Holifield radioactive ion beam facility. In both papers the measured α-decay half-lives are interpreted as indication for super-allowed α-decay in the vicinity of the doubly-magic nucleus ^{100}Sn with $Z = N = 50$. Whereas the larger experimental uncertainties in [1] allowed only to conclude "a modest enhancement of α-decay rates toward the $N = Z$ line", the latest data of [2] clearly confirm the super-allowed α-decay of ^{105}Te by comparison with the analogous α-decay of ^{213}Po. A first theoretical report by Xu and Ren [3] is based on improved folding potentials, and they find an increased α preformation factor for $N = Z$ nuclei.

The present study reanalyzes the new experimental data [1,2] using a similar model as [3] in combination with double-folding potentials which are close to the results of elastic scattering data on $N \approx Z$ data in the $A \approx 100$ mass region (^{92}Mo [4], ^{106}Cd [5], ^{112}Sn [6]). The results further confirm the super-allowed α-decay around ^{100}Sn. The systematic properties of the double-folding potentials allow a prediction of the α-decay energy of ^{104}Te and ^{108}Xe with relatively small uncertainties. However, the prediction of the α-decay half-lives has still considerable uncertainties because of the exponential dependence on the α-decay energy. In addition, α cluster properties of the nucleus $^{104}\mathrm{Te} = {}^{100}\mathrm{Sn} \otimes \alpha$ can be predicted in a similar way as in [7,8] for $^{94}\mathrm{Mo} = {}^{90}\mathrm{Zr} \otimes \alpha$. In particular, the excitation energy of the first excited 2^+ state in ^{104}Te and its decay properties by γ and α emission are calculated. These decay properties have noticeable influence on the experimental determination of the α-decay of ^{104}Te.

2 α-decay half-lives

In a semi-classical approximation the α-decay width Γ_α is given by the following formulae [9]:

$$\Gamma_\alpha = PF\frac{\hbar^2}{4\mu}\exp\left[-2\int_{r_2}^{r_3} k(r)\mathrm{d}r\right] \tag{1}$$

with the preformation factor P, the normalization factor F

$$F\int_{r_1}^{r_2}\frac{\mathrm{d}r}{2\,k(r)} = 1 \tag{2}$$

and the wave number $k(r)$

$$k(r) = \sqrt{\frac{2\mu}{\hbar^2}\,|E - V(r)|}. \tag{3}$$

μ is the reduced mass and E is the decay energy of the α-decay which was taken from the mass table of ref. [10]

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and the recent experimental results of [1,2]. The r_i are the classical turning points. For $0^+ \to 0^+$ s-wave decay the inner turning point is at $r_1 = 0$. r_2 varies around 7 fm, and r_3 varies strongly depending on the energy. The decay width Γ_α is related to the half-life by the well-known relation $\Gamma_\alpha = \hbar \ln 2 / T_{1/2,\alpha}$. Following eq. (1), the preformation factor may also be obtained as

$$P = \frac{T^{\rm calc}_{1/2,\alpha}}{T^{\rm exp}_{1/2,\alpha}}, \qquad (4)$$

where Γ_α or $T^{\rm calc}_{1/2,\alpha}$ are calculated from eq. (1) with $P = 1$. For completeness, I define the here predicted half-life for unknown nuclei as $T^{\rm pre}_{1/2,\alpha} = T^{\rm calc}_{1/2,\alpha}/P$. Further details of the model can be found in [11,12].

The potential $V(r)$ in eq. (3) is given by

$$V(r) = V_N(r) + V_C(r) = \lambda\, V_F(r) + V_C(r), \qquad (5)$$

where the nuclear potential V_N is the double-folding potential V_F multiplied by a strength parameter $\lambda \approx$ 1.1–1.3 [13]. The nuclear densities have been taken from [14] in the same parametrization as in [12] for all nuclei under study. V_C is the Coulomb potential in the usual form of a homogeneously charged sphere with the Coulomb radius R_C chosen the same as the *rms* radius of the folding potential V_F. For decays with angular momenta $L \neq 0$ an additional centrifugal potential $V_L = L(L+1)\hbar^2/(2\mu r^2)$ is used.

The potential strength parameter λ of the folding potential was adjusted to the energy of the α particle in the α emitter $(A+4) = A \otimes \alpha$. The number of nodes of the bound-state wave function was taken from the Wildermuth condition

$$Q = 2N + L = \sum_{i=1}^{4}(2n_i + l_i) = \sum_{i=1}^{4} q_i, \qquad (6)$$

where Q is the number of oscillator quanta, N is the number of nodes and L the relative angular momentum of the α-core wave function, and $q_i = 2n_i + l_i$ are the corresponding quantum numbers of the nucleons in the α cluster. I have taken $q = 4$ for $50 < Z$, $N \leq 82$, $q = 5$ for $82 < Z$, $N \leq 126$ and $q = 6$ for $N > 126$, where Z and N are the proton and neutron number of the daughter nucleus. This leads to $Q = 16$ for the nuclei above ^{100}Sn and $Q = 22$ for the nuclei above ^{208}Pb.

The results for the nuclei ^{108,109,110}Xe and ^{104,105,106}Te above the doubly-magic ^{100}Sn and for ^{216,217,218}Rn and ^{212,213,214}Po above the doubly-magic ^{208}Pb are listed in table 1. The derived preformation factors P are shown in fig. 1 as a function of ΔA_D, where ΔA_D gives the distance from a double shell closure. *E.g.*, the preformation factor for the α-decay $^{106}\mathrm{Te} \to {}^{102}\mathrm{Sn}$ can be found at $\Delta A_D = 2$ because the daughter nucleus ^{102}Sn has two nucleons above the doubly-magic ^{100}Sn. The same value of $\Delta A_D = 2$ is found for the α-decay $^{214}\mathrm{Po} \to {}^{210}\mathrm{Pb}$. Thus, a comparison between the results above $A = 100$ and above $A = 208$ can be done easily.

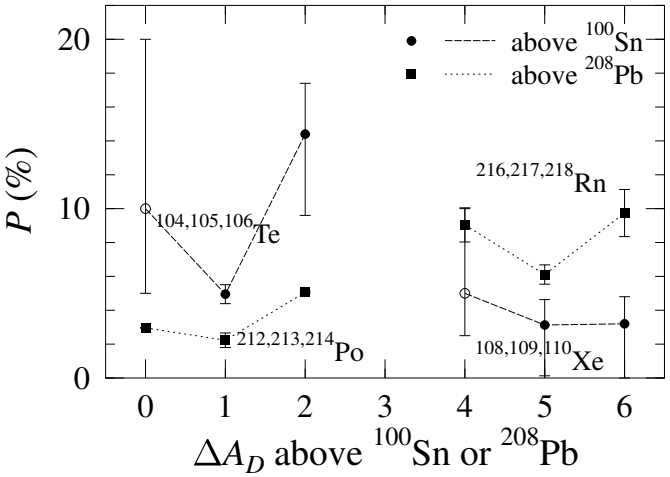

Fig. 1. Comparison of preformation factors P for the α-decays of ^{104,105,106}Te and ^{108,109,110}Xe above doubly-magic ^{100}Sn (circles) and ^{212,213,214}Po and ^{216,217,218}Rn above doubly-magic ^{208}Pb (squares), derived from eq. (4). The open circles for ^{104}Te and ^{108}Xe indicate assumed values: $P = 10\%$ for ^{104}Te and $P = 5\%$ for ^{108}Xe. The lines are to guide the eye only.

The systematic behavior of the potential parameters is one main advantage of the folding potentials. The potential strength parameter λ and the normalized volume integral per interacting nucleon pair

$$J_R = \frac{\lambda}{A_P A_T} \int V_F(r)\, \mathrm{d}^3 r \qquad (7)$$

show values around $\lambda \approx 1.10$ and $J_R \approx 303\,\mathrm{MeV\,fm^3}$ for the systems $^{100,101,102}\mathrm{Sn} \otimes \alpha$ and $^{104,105,106}\mathrm{Te} \otimes \alpha$ above $A = 100$; the variations of λ and J_R are less than 1% and allow thus extrapolations with limited uncertainties. The same range of variations of less than 1% is found for the considered systems above $A = 208$, where $\lambda \approx 1.24$ and $J_R \approx 327\,\mathrm{MeV\,fm^3}$.

The analysis of the $0^+ \to 0^+$ decays of the even-even systems is straightforward. The ground-state transitions dominate because these transitions have the maximum energy, and the decay is not hindered by an additional centrifugal barrier because $L = 0$. In both decays, $^{217}\mathrm{Rn} \to {}^{213}\mathrm{Po}$ and $^{213}\mathrm{Po} \to {}^{209}\mathrm{Pb}$, the ground-state transitions $9/2^+ \to 9/2^+$ with $L = 0$ also dominate. However, the analysis of the α-decays $^{109}\mathrm{Xe} \to {}^{105}\mathrm{Te}$ and $^{105}\mathrm{Te} \to {}^{101}\mathrm{Sn}$ requires further study.

Two α groups have been detected in the decay of $^{109}\mathrm{Xe} \to {}^{105}\mathrm{Te}$ which have been interpreted as the $L = 0$ and $L = 2$ decays from the $7/2^+$ ground state of ^{109}Xe to the $5/2^+$ ground state and $7/2^+$ first excited state in ^{105}Te [2]. From eq. (1) one calculates $T^{\rm calc}_{1/2,\alpha} = 5.71 \times 10^{-4}\,\mathrm{s}$ for the $L = 2$ ground-state decay and $T^{\rm calc}_{1/2,\alpha} = 1.42 \times 10^{-3}\,\mathrm{s}$ for the $L = 0$ decay to the first excited state, in both cases using $P = 1$. The theoretical branching is 71% for the ground-state branch and 29% for the branch to the first excited state. This is in excellent agreement with the experimental values of $(70 \pm 6)\%$ for the ground-state branch and $(30 \pm 6)\%$ for the branch to the first excited

Table 1. α-decay half-lives for nuclei above ^{100}Sn and ^{208}Pb.

Decay	$J_i \to J_f$	E (MeV)	λ	J_R (MeV fm^3)	$T_{1/2}^{\rm exp}$ or $T_{1/2}^{\rm pre}$ (s)	$T_{1/2}^{\rm calc}$ (s)	P (%)
$^{218}\text{Rn} \to ^{214}\text{Po}$	$0^+ \to 0^+$	7.263	1.2431	328.2	$(3.5 \pm 0.5) \times 10^{-2}$	3.41×10^{-3}	9.74 ± 1.39
$^{217}\text{Rn} \to ^{213}\text{Po}$	$9/2^+ \to 9/2^+$	7.887	1.2390	327.2	$(5.4 \pm 0.5) \times 10^{-4}$	3.30×10^{-5}	6.11 ± 0.57
$^{216}\text{Rn} \to ^{212}\text{Po}$	$0^+ \to 0^+$	8.200	1.2386	327.2	$(4.5 \pm 0.5) \times 10^{-5}$	4.07×10^{-6}	9.04 ± 1.01
$^{214}\text{Po} \to ^{210}\text{Pb}$	$0^+ \to 0^+$	7.834	1.2384	327.3	$(1.64 \pm 0.02) \times 10^{-4}$	8.32×10^{-6}	5.06 ± 0.06
$^{213}\text{Po} \to ^{209}\text{Pb}$	$9/2^+ \to 9/2^+$	8.536	1.2333	326.1	$(4.2 \pm 0.8) \times 10^{-6}$	9.38×10^{-8}	2.23 ± 0.43
$^{212}\text{Po} \to ^{208}\text{Pb}$	$0^+ \to 0^+$	8.954	1.2316	325.7	$(2.99 \pm 0.02) \times 10^{-7}$	8.70×10^{-9}	2.96 ± 0.02
$^{110}\text{Xe} \to ^{106}\text{Te}$	$0^+ \to 0^+$	3.885	1.0981	302.4	$\approx 4 \times 10^{-1}$ [a]	1.29×10^{-2}	≈ 3.2
$^{109}\text{Xe} \to ^{105}\text{Te}$	$7/2^+ \to 7/2^+$	4.067	1.1006	303.2	$(1.3 \pm 0.2) \times 10^{-2}$	1.42×10^{-3} [b]	≈ 3 [b]
$^{108}\text{Xe} \to ^{104}\text{Te}$	$0^+ \to 0^+$	4.65 [c]	1.099	303.4	$\approx 60\,\mu$s [c,d]	$\approx 3 \times 10^{-6}$ [d]	≈ 5 [e]
$^{106}\text{Te} \to ^{102}\text{Sn}$	$0^+ \to 0^+$	4.290	1.1026	304.5	$(6.0^{+3.0}_{-1.0}) \times 10^{-5}$	8.66×10^{-6}	$14.4^{+3.0}_{-4.8}$
$^{105}\text{Te} \to ^{101}\text{Sn}$	$5/2^+ \to 5/2^+$	4.889	1.1006	304.1	$(6.2 \pm 0.7) \times 10^{-7}$	3.07×10^{-8}	4.95 ± 0.56
$^{104}\text{Te} \to ^{100}\text{Sn}$	$0^+ \to 0^+$	5.42 [c]	1.100	304.0	≈ 5 ns [c]	$\approx 5 \times 10^{-10}$	≈ 10 [e]

[a] α-decay branch only.
[b] Branching to $7/2^+$: see sect. 2.
[c] Predicted values; see sect. 3.
[d] Huge uncertainty from unknown energy E; see sect. 3.
[e] Assumed values; see fig. 1.

state [2]. In fig. 1 I show the preformation factor P in ^{109}Xe for the $L = 0$ decay only because all the other decays in fig. 1 have the same $L = 0$.

For the α-decay $^{105}\text{Te} \to ^{101}\text{Sn}$ only one α group has been detected in [2], and an upper limit of 5% is given for other decay branches. The α-decay strength increases with increasing energy and decreasing angular momentum. If only one decay branch is observed, one may conclude that this branch corresponds to a $L = 0$ ground-state transition. Consequently, $J^\pi(^{101}\text{Sn}) = J^\pi(^{105}\text{Te}) = 5/2^+$ [2]. This is in agreement with a recent theoretical prediction [15]. The derived values for the potential strength parameter λ and the volume integral J_R fit into the systematics and thus strengthen the above tentative spin assignment.

The results in fig. 1 and table 1 confirm the super-allowed nature of α-decay near the doubly-magic ^{100}Sn. For ^{216,217,218}Rn one finds preformation values P between about 5% and 10%. Surprisingly, P slightly decreases for ^{212,213,214}Po to values between about 2% and 5% when approaching the doubly-magic daughter nucleus ^{208}Pb. For ^{109,110}Xe relatively small values of $P \approx 3\%$ are found. When approaching the doubly-magic daughter ^{100}Sn, the preformation values P show the expected behavior and increase to about 5% to 15% for ^{105,106}Te. A comparison between the preformation factors P for the Po isotopes and the Te isotopes shows that

$$P(\text{Te}) \approx 3 \times P(\text{Po}) \tag{8}$$

in agreement with the conclusions of [1,2].

3 Predicted half-lives of ^{104}Te and ^{108}Xe

The systematic behavior of the potential parameters λ and J_R in combination with the shown preformation factors P (see fig. 1) enables the extrapolation to the decays $^{108}\text{Xe} \to ^{104}\text{Te} \to ^{100}\text{Sn}$ with limited uncertainties. For the prediction of the α-decay energies I use a local potential which is adjusted to the neighboring nuclei.

The potentials for $^{105}\text{Te} = ^{101}\text{Sn} \otimes \alpha$ and $^{106}\text{Te} = ^{102}\text{Sn} \otimes \alpha$ are practically identical. From the average $J_R = 304.29\,\text{MeV fm}^3$ one obtains the α-decay energy of ^{104}Te $E = 5.354\,\text{MeV}$, whereas a linear extrapolation yields a slighly weaker potential $J_R = 303.76\,\text{MeV fm}^3$ and slightly higher energy $E = 5.481\,\text{MeV}$. Combining these results, a reasonable prediction of the α-decay energy of ^{104}Te is $E = 5.42 \pm 0.07\,\text{MeV}$.

From the lower decay energy $E = 5.354\,\text{MeV}$ one obtains $T_{1/2,\alpha}^{\rm calc} = 7.87 \times 10^{-10}\,\text{s}$ from eq. (1) with $P = 1$; the higher decay energy yields $T_{1/2,\alpha}^{\rm calc} = 3.13 \times 10^{-10}\,\text{s}$. The uncertainty of the α-decay energy of about 70 keV translates to an uncertainty in the calculated half-life of about a factor of 1.5. For a prediction of the α-decay half-life one has to find a reasonable assumption for the preformation factor P. Following the pattern of P in fig. 1, I use $P = 10\%$ with an estimated uncertainty of a factor of two. Combining the above findings, the predicted half-life of ^{104}Te is $T_{1/2,\alpha}^{\rm pre} = 5\,\text{ns}$ with an uncertainty of about a factor three. The uncertainty of the predicted half-life is composed of similar contributions for the unknown α-decay energy and the assumed preformation factor P.

The potentials for $^{109}\text{Xe} = ^{105}\text{Te} \otimes \alpha$ and $^{110}\text{Xe} = ^{106}\text{Te} \otimes \alpha$ change by about $1\,\text{MeV fm}^3$; this is still very similar, but not as close as in the above $^{105}\text{Te} = ^{101}\text{Sn} \otimes \alpha$ and $^{106}\text{Te} = ^{102}\text{Sn} \otimes \alpha$ cases. Repeating the above procedure, one finds the α-decay energy $E = 4.792\,\text{MeV}$ from the average $J_R = 302.82\,\text{MeV fm}^3$ and $E = 4.506\,\text{MeV}$ from the extrapolated $J_R = 303.96\,\text{MeV fm}^3$. The calculated half-lives using $P = 1$ are $T_{1/2,\alpha}^{\rm calc} = 7.40 \times 10^{-7}\,\text{s}$ for the higher energy $E = 4.792\,\text{MeV}$ and $T_{1/2,\alpha}^{\rm calc} = 1.18 \times 10^{-5}\,\text{s}$ for the lower energy $E = 4.506\,\text{MeV}$. Combining these

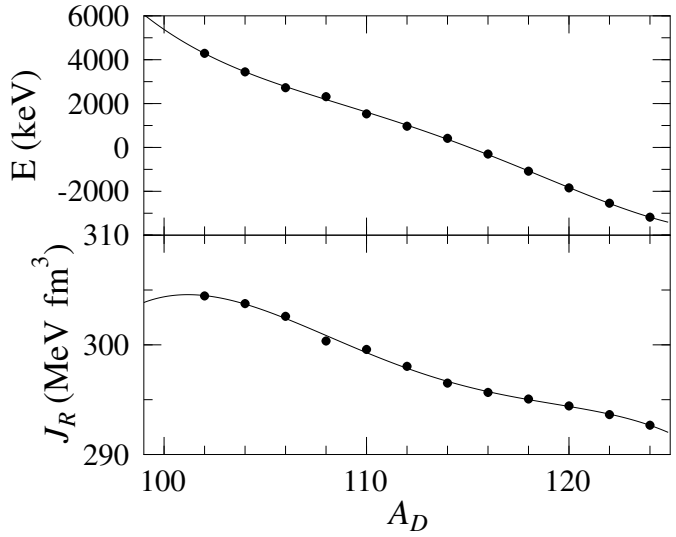

Fig. 2. Volume integral J_R and energy E in dependence on the mass number A_D from ^{102}Sn to ^{124}Sn (See text).

results, the α-decay energy is $E = 4.65 \pm 0.15$ MeV. Together with a preformation factor of about $P = 5\%$ the α-decay half-life is predicted to be of the order of 100 μs. However, the uncertainty of the decay energy of 150 keV leads to an uncertainty in the half-life of a factor of 4; thus it is impossible to predict the α-decay half-life of ^{108}Xe better than this uncertainty.

It is interesting to compare the predictions for the α-decay properties of ^{104}Te with the results of [3]. In [3] the α-decay energy is linearly extrapolated from the neighboring even-even Te isotopes ^{106,108,110}Te leading to $E = 5.053$ MeV. I have repeated this procedure for the Te isotopes ^{106}Te to ^{126}Te. The α-decay energies and derived volume integrals J_R are shown in fig. 2. For an extrapolation to the α-decay of ^{104}Te I have fitted the data in fig. 2 using a polynomial

$$E(A_D) = \sum_{i=0}^{n} a_i \, (A_D - 100)^i \qquad (9)$$

and a corresponding formula for the volume integral J_R. It has turned out that the reduced χ^2 of the fit improves when one increases the number n up to $n = 4$; no further significant improvement is found for larger values of n. These fourth-order polynomials for E and J_R are shown as lines in fig. 2. The resulting numbers for $A_D = 100$, *i.e.* the $^{104}\text{Te} \rightarrow {}^{100}\text{Sn}$ α-decay, are $E = 5.379$ MeV and $J_R = 304.4$ MeV fm^3 which is within the error bars of the values derived above from the neighboring potentials.

Because of the higher α-decay energy derived in this work, the α-decay half-life of ^{104}Te is about a factor of 10 shorter compared to the predictions of [3]. Experimental data are required to distinguish between the predictions of this work and ref. [3].

The results for ^{108}Xe roughly agree with the predictions in [3]: Xu *et al.* predict the α-decay energy $E = 4.44$ MeV compared to $E = 4.65 \pm 0.15$ MeV in this work, and the predicted half-life in [3] is between 150 and 290 μs which should be compared to the predicted half-life of $T^{\rm pre}_{1/2,\alpha} = 236\,\mu$s derived from the lower limit $E = 4.5$ MeV of the energy with $P = 5\%$.

Table 2. Comparison of α-decay energies from a local extrapolation using folding potentials (this work) to predictions of global mass formula [16–20]. All energies are given in MeV.

	Exp. or this work	FRDM [17]	HFB-1 [18,16]	HFB-2 [19]	DZ [20]
^{104}Te	5.42 ± 0.07^a	6.12	4.85	4.68	5.24
^{105}Te	4.89	6.31	4.91	4.28	4.91
^{106}Te	4.29	6.01	4.72	4.16	4.60
^{108}Xe	4.65 ± 0.15^a	5.53	4.69	4.38	4.93
^{109}Xe	4.22^b	4.81	4.23	4.03	4.62
^{110}Xe	3.89	4.61	3.60	3.71	4.33

^{a} Predicted from folding potential.
^{b} From ground state in ^{109}Xe to ground state in ^{105}Te.

4 Comparison to mass formulae

The α-decay energies of the folding calculation may be compared to predictions from global mass formulae. Here I restrict myself to the three selected mass formulae of the so-called Reference Input Parameter Library RIPL-2 of the IAEA [16] which are the Finite Range Droplet Model (FRDM) [17], the Hartree-Fock-Bogoliubov (HFB) method [18] in the versions of [16] and its latest update [19], and the simple 10-parameter formula of Duflo and Zuker (DZ) [20]. The results are listed in table 2.

The FRDM predictions seem to overestimate the experimental α-decay energies slightly, especially when approaching the doubly-magic core ^{100}Sn. The predictions of HFB-1 and HFB-2 are close to the experimental values, and also the simple 10-parameter parametrization DZ is in reasonable agreement with the data. The predictions from the folding potential calculation for ^{104}Te and ^{108}Xe are close to the average values of the above global mass models [16–20].

5 Accuracy of semi-classical half-lives

The results which are presented in table 1 and fig. 1 have been obtained using the semi-classical approximation of eq. (1) for the decay width Γ_α. From a fully quantum-mechanical analysis the decay width Γ_α is related to the energy dependence of a resonant scattering phase shift $\delta_L(E)$ by

$$\delta_L(E) = \arctan \frac{\Gamma_\alpha}{2(E_R - E)} \, . \qquad (10)$$

In practice, it is difficult to determine widths of the order of 1 μeV at energies of the order of several MeV because of numerical problems. For the system $^{104}\text{Te} = {}^{100}\text{Sn} \otimes \alpha$ such an analysis is possible at the limits of numerical stability.

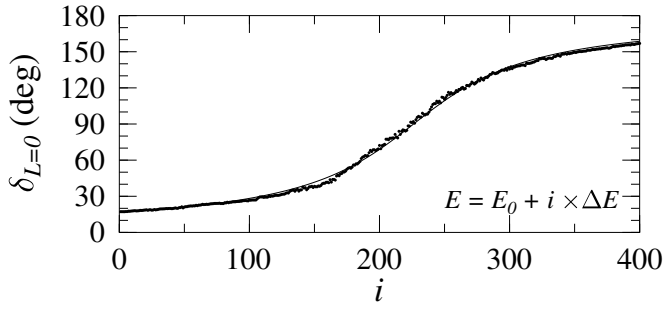

Fig. 3. Phase shift δ_L for the $L = 0$ partial wave for the system $^{104}\text{Te} = {}^{100}\text{Sn} \otimes \alpha$. The derived width from eq. (10) is $\Gamma = 1.36 \times 10^{-12}$ MeV. Note the extremely small stepsize of the calculation of $\Delta E = 1.0 \times 10^{-14}$ MeV! See text for details.

In fig. 3 the resonant behavior of the s-wave phase shift $\delta_{L=0}(E)$ is shown around the resonance energy $E_R = 5.481$ MeV which is obtained in the potential with $J_R = 303.76$ MeV (see sect. 3). The dots are obtained from solving the Schrödinger equation at $E = E_0 + i \times \Delta E$ with $E_0 = 5.481305851985$ MeV and $\Delta E = 10^{-14}$ MeV. The full line is a fit of data using eq. (10) where the resonance energy E_R and the width Γ_α have been adjusted. This yields $\Gamma_\alpha = 1.36\,\mu$eV and a corresponding half-life of $T^{\rm calc}_{1/2,\alpha} = 0.336$ ns. The semi-classical approximation in eq. (1) gives $T^{\rm calc}_{1/2,\alpha} = 0.313$ ns which is about 8% lower than the value from the fully quantum-mechanical calculation.

The validity of the semi-classical approximation for Γ_α in eq. (1) is confirmed for the α-decay of ^{104}Te by the above analysis of the scattering phase shift $\delta_L(E)$ with an uncertainty of less than 10%. For two other nuclei (^{8}Be and ^{212}Po) the semi-classical approximation deviates by about 30% from the fully quantum-mechanical value. In all cases the semi-classical half-life is slightly shorter than the fully quantum-mechanical result.

In a detailed study on proton-decay half-lives of proton-rich nuclei [21] it has been shown that the semi-classical approximation agrees within about ±10% with the result of a direct calculation of the transition amplitude using the distorted-wave Born approximation (DWBA) formalism. Surprisingly, the agreement between the quantum-mechanical DWBA calculation and the semi-classical result becomes worse in [21] when an improved normalization factor from eq. (25) of [21] is used compared to the simple normalization factor in eq. (24) of [21] or eq. (2) in this work: For the case of ^{104}Te, the α-decay half-life in the semi-classical calculation changes from 0.313 ns using eq. (2) to 0.231 ns using eq. (25) of [21]; thus, the findings in [21] are confirmed.

6 Properties of $^{104}\text{Te} = {}^{100}\text{Sn} \otimes \alpha$

From the given potential of the system $^{104}\text{Te} = {}^{100}\text{Sn} \otimes \alpha$ it is not only possible to determine the α-decay half-life of the ground state. Following the formalism in [22], energies and electromagnetic decay properties of excited states in ^{104}Te can be predicted.

Table 3. Excitation energies $E_x = E - E(0^+)$ of excited states in $^{104}\text{Te} = {}^{100}\text{Sn} \otimes \alpha$ with $Q = 16$, 17, and 18.

J^π	Q	N	L	λ	E (keV)	E_x (keV)
0^+	16	8	0	1.1005	5354.2	0.0
2^+	16	7	2	1.0915	6003.5	649.3
4^+	16	6	4	1.0825	6739.6	1385.4
6^+	16	5	6	1.0735	7565.2	2211.0
8^+	16	4	8	1.0645	8477.2	3123.0
10^+	16	3	10	1.0555	9469.2	4115.0
12^+	16	2	12	1.0465	10543.2	5189.0
14^+	16	1	14	1.0375	11731.2	6377.0
16^+	16	0	16	1.0285	13097.1	7742.9
1^-	17	8	1	1.0960	10951.4	5597.2
0^+	18	9	0	1.1005	≈ 15 MeV[a]	≈ 10 MeV[a]

[a] Very broad.

The ground-state wave function of ^{104}Te is characterized by $Q = 2N + L = 16$, see eq. (6). Further members of this $Q = 16$ band are expected with $J^\pi = 2^+, 4^+, \ldots, 16^+$. It has been observed that the potential strength parameter λ has to be varied slightly to obtain an excellent prediction of the excitation energies:

$$\lambda(L) = \lambda(L = 0) - c \times L \qquad (11)$$

with the constant $c \approx (3\text{–}5) \times 10^{-3}$ for neighboring $N = 50 \otimes \alpha$ nuclei $^{94}\text{Mo} = {}^{90}\text{Zr} \otimes \alpha$ [7,8], $^{93}\text{Nb} = {}^{89}\text{Y} \otimes \alpha$ [23], neighboring $Z = 50$ nuclei $^{116}\text{Te} = {}^{112}\text{Sn} \otimes \alpha$, and the systems $^{20}\text{Ne} = {}^{16}\text{O} \otimes \alpha$ [24], $^{44}\text{Ti} = {}^{40}\text{Ca} \otimes \alpha$ [13], and $^{212}\text{Po} = {}^{208}\text{Pb} \otimes \alpha$ [25].

For the following analysis I adopt $\lambda = 1.1005$ which corresponds to $J_R = 304.29\,\text{MeV fm}^3$ from the average of the two neighboring systems $^{105,106}\text{Te} = {}^{101,102}\text{Sn} \otimes \alpha$ and $c = (4.5 \pm 0.3) \times 10^{-3}$ from the neighboring nuclei ^{93}Nb, ^{94}Mo, and ^{116}Te above $N = 50$ or $Z = 50$ cores. Because the predicted excitation energies $E_x = E - E(0^+)$ (see table 3) are relative to the ground-state energy, the excitation energies do not change significantly when one varies $\lambda(L = 0)$ or J_R within the given uncertainties.

The first excited 2^+ state in ^{104}Te is found at $E_x = 649$ keV. From the uncertainty of the constant c in eq. (11) one can derive a very small uncertainty for the potential strength $\lambda(L = 2)$ and a resulting uncertainty of about 40 keV for the excitation energy E_x for the first 2^+ state. Somewhat larger uncertainties are found for $\lambda(L > 2)$; consequently, the uncertainty of the predicted excitation energies increases up to about 400 keV for the 16^+ state at $E_x = 8.55$ MeV.

In addition, the 1^- and 0^+ band heads of the bands with $Q = 17$ and $Q = 18$ are predicted at energies around $E_x = 5.60$ MeV and about 10 MeV. The 0^+ state is very broad. It is difficult to estimate the uncertainty of the predicted energies of the 1^- and 0^+ states with $Q = 17$ and $Q = 18$ because usually the potential strength has to be slightly readjusted to obtain a good description of such bands. A rough estimate for the uncertainty is about 1 MeV which corresponds to an uncertainty of about 2% for the potential strength parameter λ.

Following the formalism of ref. [22], reduced transition strengths of 10.1 W.u., 14.0 W.u., and 14.1 W.u. are calculated for the $2^+ \to 0^+$, $4^+ \to 2^+$, and $6^+ \to 4^+$ transitions in ^{104}Te. The corresponding radiation widths Γ_γ are slightly larger than the direct α-decay widths from the excited states in ^{104}Te to the ground state in ^{100}Sn. The γ-decay branching ratio

$$b_\gamma = \frac{\Gamma_\gamma}{\Gamma_\gamma + \Gamma_\alpha^{\rm pre}} \tag{12}$$

is between 86% and 93% for the 2^+ state, between 62% and 76% for the 4^+ state, and between 48% and 62% for the 6^+ state. This is an extremely important result for future experiments! If the γ-decay branch b_γ of the first 2^+ state were small (*e.g.*, of the order of a few per cent), it would be extremely difficult to produce ^{104}Te in its ground state because ^{104}Te produced in excited states could directly decay to the ^{100}Sn ground state by α emission.

It is interesting to note that the predicted branchings b_γ are not very sensitive to the predicted excitation energy. *E.g.*, if the excitation energy of the first excited 2^+ state in ^{104}Te is $E_x = 1\,$MeV, the radiation width Γ_γ increases with E_γ^5 by a factor of about 9 and the width Γ_α increases by a factor of about 8 because of the reduced Coulomb barrier. Thus, b_γ values close to unity are very likely. Consequently, a direct production reaction like *e.g.* ^{50}Cr(^{58}Ni, $4n$)^{104}Te similar to the experiment in [1] should be feasible. However, only the indirect production via the α-decay of ^{108}Xe in a reaction like, *e.g.*, ^{54}Fe(^{58}Ni, $4n$)^{108}Xe similar to [2] ensures the production of ^{104}Te in its ground state.

7 Conclusions

The systematic properties of folding potentials provide a powerful tool for the analysis of the system $^{104}\text{Te} = {}^{100}\text{Sn} \otimes \alpha$ above the doubly-magic ^{100}Sn core. In particular, α-decay energies and half-lives can be predicted with relatively small uncertainties. The predicted α-decay energy for ^{104}Te is $E = 5.42 \pm 0.07\,$MeV, and the corresponding half-life is $T_{1/2,\alpha}^{\rm pre} = 5\,$ns with an uncertainty of a factor of three.

Excitation energies and decay properties of the members of the $Q = 16$ rotational band in ^{104}Te are calculated, and the predicted values have small uncertainties. For the first excited 2^+ state in ^{104}Te one obtains $E_x = 650 \pm 40\,$keV. The γ-decay strength to the ground state in ^{104}Te is about 10 Weisskopf units. The corresponding radiation width Γ_γ is about a factor of 10 larger than the α-decay width Γ_α to the ground state in ^{100}Sn.

The finding that Γ_γ is larger than Γ_α for excited states in ^{104}Te is important for the experimental production of ^{104}Te in its ground state and the measurement of the α-decay half-life of ^{104}Te. The condition $\Gamma_\gamma > \Gamma_\alpha$ allows to use reactions which produce ^{104}Te in excited states because these states preferentially decay to the ^{104}Te ground state. However, only the indirect production of ^{104}Te via the α-decay of ^{108}Xe safely guarantees that ^{104}Te is produced in its ground state.

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